

Test Kaleidoscope Mathematics (WPMA18002)

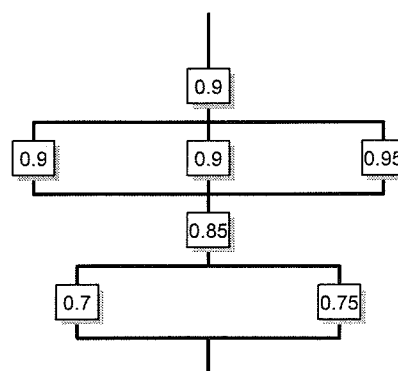
Statistics and Probability

Tuesday November 6, 14:00-15:00, MartiniPlaza L Springerlaan 2, 9727

- During the exam it is allowed to use a simple calculator such as the CASIO FX-82 or TI30.
- Give in all cases the reasoning leading to your answer.
- Provide each page with your name and student number.
- Use a sufficient number of decimals to make your computations unambiguously.
- This test consists of 6 exercises printed on 2 separate pages. We wish you lot's of success with its completion!

1. **Pair of Dice.** 10 A pair of dice is rolled 24 times. Compute the probability that at least one double 6 is obtained.

2. **Advanced circuit.** 20 The circuit in the figure at the right hand side operates only if there is a path of functional devices from the top to the bottom. The probability that each device functions is as indicated. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?



see next page

3. **Purchasing Microchips.** [20] An important component of your (desktop or laptop) personal computer (PC) is a microchip. The proportions of microchips that a certain PC manufacturer purchases from the suppliers S_1 , S_2 , and S_3 is .15, .30, and .55, respectively. It is known that the proportions of defective microchips produced by the three suppliers are 0.002, 0.005, and 0.001, respectively. If a single PC microchip failure is observed, which supplier is the most likely responsible?

4. **Transition Matrix.** Suppose we have the following probability transition matrix

$$P = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix}$$

for a three state Markov process

(a) [20] Determine the stationary distribution of the Markov chain.

Hint: This is the distribution π , represented a row vector, such that $\pi P = \pi$.

(b) [5] Is the corresponding Markov chain irreducible?

5. **General 2 by 2 Transition Matrix.** Consider a general two-state Markov chain with probability transition matrix

$$P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}, 0 < a < 1, 0 < b < 1.$$

(a) [20] Show that the n -step transition probability matrix P^n is given by

$$P^n = \frac{1}{a+b} \left\{ \begin{bmatrix} b & a \\ b & a \end{bmatrix} + (1-a-b)^n \begin{bmatrix} a & -a \\ -b & b \end{bmatrix} \right\}.$$

Hint: It is sufficient to show that this holds for P^1 and $P^{n+1} = P^n P$.

(b) [5] Determine the limiting transition probability matrix, that is $\lim_{n \rightarrow \infty} P^n$.

End of test.